
OPEN
DISCUSSION PAPERS IN
ECONOMICS

# Surplus Value and the Kalecki Principle in Marx's Reproduction Schema

Andrew B Trigg

March 2001

**NUMBER 40**

***Copies may be obtained from**:*
Economics Department
Faculty of Social Sciences
The Open University
Walton Hall  Milton Keynes  MK7 6AA
Telephone:  01908 654437
Email:  Socsci-economics-support-list@open.ac.uk
Fax:  01908 654488

This series is registered under
ISSN 1753-2590 (Print)
ISSN 1753-2604 (Online)

**Economics Research at The Open University**

Throughout the 1990s, The Open University has been developing its research capacity in economics. Economists at the OU comprise a lively and expanding group with a wide set of interests ranging from development policy to decision theory, from Marxist theories of profit to libertarian foundations of environmental policy and from econometric analysis of large data sets through institutional economics to the use of case-studies in policy formation. Nearly a 1000 students from around the world register each year to study economics courses and their needs, together with the multi-disciplinary nature of social science at the university, shape out research. Through a variety of personal and group research projects, our work makes a strong contribution to areas like business, public policy and even philosophy where sharply focused analysis can inform decision-making as well as contribute to scientific progress.

In 1999, approximately £250,000 million worth of externally funded grants (3 from the ESRC) were held by discipline members, some of whom also act as consultants to national and international bodies. Approximately half a dozen students are currently reading for doctorates with members of the discipline and we are always interested in proposals from colleagues or potential students who would like to do research with us.

Some of the journals in which discipline members have published include: *Annals of Operations Research, Economic Journal, Economica, Economics and Philosophy, Feminist Economics, Feminist Review, International Journal of the Economics of Business, International Journal of Industrial Organisation, Journal of Economic Issues, Journal of Economic Psychology, Journal of the History of Ideas, Journal of Social Policy, Local Government Studies, The Locke Newsletter, Open Learning, Oxford Economic Papers, Public Policy and Administration, Radical Statistics, Revue d' Économie Politique, Risk Decision and Policy, Structural Change and Economic Dynamics, Technovation* and *Theory and Decision.*

The papers contain results of economic research which are the sole responsibility of the authors. Opinions expressed in these papers are hence those of the authors and do not necessarily reflect views of the University.

# Surplus Value and the Kalecki Principle in Marx's Reproduction Schema

Andrew B Trigg

March 2001

The Economics Discipline
The Faculty of Social Sciences
The Open University
Walton Hall
Milton Keynes MK7 6AA
A.B.Trigg@open.ac.uk

# 1 Introduction

A well-known interpretation of Marx's reproduction schema identifies the role played by the 'Kalecki principle', or Widow's Curse, that capitalists earn what they spend. As Marx writes in Capital Volume II: 'In point of fact, paradoxical as it may seem at the first glance, the capitalist class itself casts into circulation the money that serves towards the realization of the surplus-value contained in its commodities' (Marx, 1978, p. 409). In their particularly extensive analyses of the reproduction schema both Reuten (1998, 200) and Sardoni (1989, 212) argue that for Marx profits are determined by capitalist expenditure outlays.

There are two main ways in which this interpretation of the reproduction schema is underdeveloped. First, although Kalecki (1968, 459) claims that his model is 'fully in the Marxian spirit' he did not examine the direct relationship between his approach and Marx's original text. Sardoni (1989) has provided perhaps the most concerted effort to make this connection but does not engage directly with Marx's numerical examples. Second, coming from the other extreme, Reuten (1998) provides a most systematic and detailed exploration of Marx's original tables, giving special mention to the Kalecki principle, but without providing a direct connection to Kalecki's analytical model of the reproduction schema.

The contribution of this paper is to provide a detailed analysis of the role of the Kalecki principle in Marx's reproduction schema. Using Marx's original tables, in the first part of the paper a number of steps are followed to make the transition to Kalecki's model. This model is shown to provide a particular ex post interpretation of Marx's tables. A key problem with this interpretation is that it obscures the dassical role of surplus value in the reproduction schema. This has led, perhaps unfairly, to Kalecki being described in some circles as 'non-Marxist' (Freeman and Carchedi 1996, xii ). In the second part of the paper a different interpretation of the reproduction schema is offered using the Leontief input-output framework. On this interpretation, both the Kalecki principle and the role of surplus value can be succinctly modelled in the context of Marx's original reproduction schema.

# 2 Kalecki's Interpretation of Marx's Reproduction Schema

The most developed of the expanded reproduction schema are referred to by Marx as 'schema (B)' of the 'First Example' in Section 3 of Chapter 21, Capital Volume II (Marx 1978, pp. 586-589). In the analysis that follows we will start with this two-sector numerical example and by a number of steps show how it relates to the three-sector model developed by Kalecki.

Table 1 reports the first two years of schema, with Department I producing capital goods and Department II consumption goods.1 In each department $i$ the total value of production $(X_i)$ is made up of constant capital $(C_i)$, variable capital $(V_i)$ and surplus value $(S_i)$. The rate of surplus value is assumed to be 100 per cent in each department, and £1 of output is assumed equal to a unit of labour.

**Table 1 Marx's Two-Sector Reproduction Schema**

| Year 1 | $C_i$ | $V_i$ | $S_i$ | $X_i$ |
|---|---|---|---|---|
| Dept. I | 4000 | 1000 | 1000 | 6000 |
| Dept. II | 1500 | 750 | 750 | 3000 |
| | 5500 | 1750 | 1750 | 9000 |
| Year 2 | $C_i$ | $V_i$ | $S_i$ | $X_i$ |
| Dept. I | 4400 | 1100 | 1100 | 6600 |
| Dept. II | 1600 | 800 | 800 | 3200 |
| | 6000 | 1900 | 1900 | 9800 |

Marx assumes that capitalists in department I invest a half of their surplus value for accumulation in the next year. This invested surplus amounts to 500 units that are distributed in year 1 between 400 additional units of constant capital and 100 additional units of variable capital. The new volumes of 4400 constant capital and 1100 variable capital in year 2 show that the organic composition of capital, the proportion between constant and variable capital, is maintained at a 4:1 ratio. By also maintaining the composition of capital in department II at its original 2:1 ratio a new position of balance between the two departments is established.

A first step in the introduction of Kalecki's model to the reproduction schema is to explicitly show how the elements of surplus value are allocated. Table 2 distinguishes between capitalists' consumption $(u_i)$, incremental changes in constant capital $(dC_i)$ and changes in variable capital $(dV_i)$. In department I, for example, the half of surplus value that capitalists do not invest is allocated to 500 units of their personal consumption. Capitalists consume 1100 units in total.

**Table 2 The Allocation of Surplus Value in the Two-Sector Scheme**

| Year 1 | $C_i$ | $V_i$ | $u_i$ | $dC_i$ | $dV_i$ | $X_i$ |
|---|---|---|---|---|---|---|
| Dept. I | 4000 | 1000 | 500 | 400 | 100 | 6000 |
| Dept. II | 1500 | 750 | 600 | 100 | 50 | 3000 |
| | 5500 | 1750 | 1100 | 500 | 150 | 9000 |

Following Kalecki (1968, 459), the reproduction scheme can be further disaggregated by dividing the activity of department II, producing consumption goods, into a new department 2 producing capitalists' consumption goods and a department 3 producing wage goods. The numbers in Table 3 provide an illustration of how Marx's scheme could be looked at from Kalecki's perspective. Note that with department 2 producing 1100 units of capitalists' consumption goods, and department 3 producing 1900 of wage goods, the combined total output of 3000 units is the same as the original output of department II in Marx's scheme. Similarly, department 1 produces exactly the same output (6000 units) as department I in the original scheme. Table 3 can be seen as a decomposition of Marx's scheme to provide a more detailed analysis of the structure of consumption.

**Table 3 Ex Ante Three-Sector Reproduction Scheme**

| Year 1 | $C_i$ | $V_i$ | $u_i$ | $dC_i$ | $dV_i$ | $X_i$ |
|---|---|---|---|---|---|---|
| Dept. 1 | 4000 | 1000 | 500 | 400 | 100 | 6000 |
| Dept. 2 | 550 | 275 | 220 | 36⅔ | 18⅓ | 1100 |
| Dept. 3 | 950 | 475 | 380 | 63⅓ | 31⅔ | 1900 |
| | 5500 | 1750 | 1100 | 500 | 150 | 9000 |

The reproduction schemes shown thus far can be characterized as showing the ex ante production of year 1 (see Desai 1979, 149; Reuten 1998, 225). At the start of the year capitalists consume 5500 units of constant capital in total to produce 6000 units of output of constant capital. There is an ex ante imbalance between these two quantities, and also between quantities of consumption goods produced and consumed. In order to ensure ex post balance, at the end of year 1, the additional units of constant $(dC_i)$ and variable $(dV_i)$ capital set aside for future production can be grouped together with the ex ante volumes of capital consumed at the start of the period (Table 4).2 Department 1, for example, has constant capital of 4400 units of constant capital at the end of the period, made up of the original 4000 consumed and the additional 400 required for production in the next period. Similarly variable capital is now 1100 units, made up of the original 1000 units and the new 100 inputs of variable capital. The new ex post categories of constant and variable capital are referred to in Table 4 as $C_i^*$ and $V_i^*$ respectively.

**Table 4 Ex Post Three-Sector Reproduction Scheme**

| Year 1 | $C_i^*$ | $V_i^*$ | $u_i$ | $X_i$ |
|---|---|---|---|---|
| Dept. 1 | 4400 | 1100 | 500 | 6000 |
| Dept. 2 | 586⅔ | 293⅓ | 220 | 1100 |
| Dept. 3 | 1013⅓ | 506⅔ | 380 | 1900 |
| | 6000 | 1900 | 1100 | 9000 |

A final re-arrangement of the categories in Marx's numerical scheme can be achieved by introducing a different way of looking at profits. For Marx, profits in each department are specified as the surplus value left after accounting for ex ante inputs of constant and variable capital $(S_i = X_i - C_i - V_i)$. However, for Kalecki profits in each department $(P_i^*)$ are the total value left after accounting for ex post variable capital $(P_i^* = X_i - V_i^*)$. Kalecki is concerned with gross profits before deductions of items such as depreciation that form part of constant capital. This gross definition of profits can be applied to the reproduction scheme by simply adding the constant capital components of Table 4 to the components for capitalists' consumption. In Department 1, for example, 4400 units of constant capital are added to 500 units of capitalists' consumption, resulting in 4900 of gross profits. This result is shown in Table

5, which gives a numerical demonstration of Kalecki's categories of wages $(V_i^*)$ and profits $(P_i^*)$. (The full algebraic structure of the three-sector schema is laid out in the Appendix).

**Table 5 Kalecki's Interpretation of the Three-Sector Scheme**

| Year 1 | $V_i^*$ | $P_i^*$ | $X_i$ |
|---|---|---|---|
| Dept. 1 | 1100 | 4900 | 6000 |
| Dept. 2 | 293⅓ | 806⅔ | 1100 |
| Dept. 3 | 506⅔ | 1393⅓ | 1900 |
| | 1900 | 7100 | 9000 |

Having re-formulated Marx's categories and re-arranged the reproduction scheme, along the lines suggested by Kalecki, a key result is established. Table 5 shows that department 3 produces a surplus of $1393\frac{1}{3}$ wage goods, and these are sold to workers in the other two departments $(1393\frac{1}{3} = 1100 + 293\frac{1}{3})$. Expressing this identity in algebraic terms:

$$P_3^* = V_1^* + V_2^* \qquad (1)$$

Following Kalecki (1968, 460), adding $P_1^* + P_2^*$ to both sides of equation (1) yields

$$P_1^* + P_2^* + P_3^* = P_1^* + V_1^* + P_2^* + V_2^* \qquad (2)$$

and hence:

$$P^* = X_1 + X_2 \qquad (3)$$

This is an ex post identity between total profits $(P^*)$ and the economy's output of capital goods $(X_1)$ and capitalists' consumption goods $(X_2)$. Kalecki poses the key question as to how we should interpret this identity? Are expenditures upon capital goods and capitalists' consumption goods determined by profits, or are profits determined by these expenditures? He argues that 'capitalists can decide how much they will invest and consume next year, but they cannot decide how much they shall sell and profit' (Kalecki 1968, 461). It is the money expenditures by capitalists upon consumption and investment that generate the resultant volume of profits.

Cartelier (1996, 217) has linked this so-called 'Kalecki Principle', that capitalists earn what they spend, to the circulation of money. 'As a result of their ability to initiate circulation entrepreneurs, as a whole, more or less have the power to determine their income'. Moreover, he argues that '…the Kalecki Principle does not contradict the Classical view which makes profit equal to the value of surplus'.

Key passages in Marx's writings, that demonstrate the role of the Kalecki Principle in relation to the circulation of money, are in Chapter 17 of Capital Volume II (see Sardoni, 1989, 211). Starting with the case of simple reproduction Marx considers the circulation of money using the example of an individual capitalist. 'During the first year he advances a money capital of £5,000, let us say, in payment for means of production (£4,000) and for labour-power (£1,000)' (Marx 1978, 409). At a 100 per cent rate of surplus value it can be assumed that £1,000 of surplus-value is appropriated. The problem is that the capitalist advances £5,000, which can be referred to as $M$, but receives back £6,000, the realized amount $M'$. Focusing upon the difference between the two amounts $(M'-M)$ Marx poses the question, 'where does this money come from?' (ibid., 407).

The simple answer to this question is that the extra money is provided by the unproductive personal expenditure of the capitalist. The capitalist consumes the same £1,000 as the volume of surplus value. This '£1,000 is converted into money with the money that he threw into circulation not as capitalist, but as consumer, i.e. did not advance, but actually spent' (ibid., 410). Moreover, this consumption is financed out of the capitalist's own money hoard: it 'means nothing more than that he has to cover his individual consumption for the first year out of his own pocket…' (ibid., 409).

Marx generalizes this key role for unproductive expenditure to the capitalist class as a whole:

'It was assumed in this case that the sum of money that the capitalist casts into circulation to cover his individual consumption until the first reflux of his capital is exactly equal to the surplus-value that he produces and hence has to convert into money. This is obviously an arbitrary assumption in relation to the individual capitalist. But it must be correct for the capitalist class as a whole, on the assumption of simple reproduction. It simply expresses the same thing as this assumption implies, namely that the entire surplus-value is unproductively consumed…' (ibid., 410).

Since there is no expansion of the capital stock under simple reproduction, all surplus value is directed to unproductive expenditure, but at the same time capitalists enable this mass of surplus value to be realized by financing unproductive expenditure out of money hoards.

The case of expanded reproduction, as considered in Tables 1 to 5 above, 'does not offer any new problems with respect to money circulation' (ibid., 418). The difference is that part of the additional money cast into circulation $(M'-M)$ now consists of money capital advance for productive purposes. (The other part consists of the money cast into circulation for purposes of unproductive expenditure by capitalists, as before in the case of simple reproduction). 'As far as the additional money capital is concerned, that required for the function of the increased productive capital, this is supplied by the portion of realized surplus-value that is cast into circulation by the capitalists as money capital, instead of as the money form of revenue' (ibid., 418). Under expanded reproduction, surplus value is clearly realized by capital investment and capitalists' consumption. Hence for Sardoni (1989, 214): 'Capitalists' profits therefore now depend on their consumption and investment expenditure, just as in Kalecki's analysis'. There is strong evidence for the Kalecki principle, that capitalists earn what they spend, operating in Marx's analysis of expanded reproduction.

The problem, however, as we have seen in the above manipulations of the reproduction schema, is that Kalecki's demonstration (of the Kalecki principle) requires a gross definition of profits, before deductions, that is different from Marx's category of surplus value. The Kalecki principle has not been precisely demonstrated in the context of Marx's reproduction schema, in which surplus value is the key category of analysis.3 To apply the Kalecki principle directly to Marx's schema, attention can be focused on an important difference between Marx and Kalecki about the way in which investment is specified. Whereas for Kalecki investment is associated specifically with capital goods produced by the capital goods producing department, for Marx, as shown in the above example, investment (accumulation) is directed to both constant and variable goods – goods produced by both the capital and wage goods producing departments. Although Sardoni (1989, 211) mentions these different specifications of investment in his comparison of Marx and Kalecki, their importance is not highlighted. To demonstrate the importance of this difference, in the next part of the paper Leontief's input-output framework can be used to model the final demand of each department of production, such that investment demand cuts across departments.

## 3 The Kalecki Principle in an Input-Output Framework

A Leontief input-output table can be constructed by re-expressing the elements of Marx's numerical reproduction schema. Table 6(a) is a numerical representation of the three-sector reproduction scheme considered previously in Tables 3 to 5. Elements of this table can be read along the rows as outputs of a particular sector, or column-wise as inputs to that sector.4 For example, department 3 produces outputs of 1000 wage goods for department 1, 275 for department 2 and 475 for itself. Reading column-wise, department 3 uses inputs of 950 constant capital from department 1 and 475 inputs of wage goods from itself. On this interpretation the surplus value elements $(S_i)$ are viewed as inputs of value added to each department. Final demand is made up of the total amounts of new investment in constant capital $(dC)$ and variable capital $(dV)$, and the total personal consumption of capitalists $(u)$.5 Taking these elements of the table into account, inputs and outputs are balanced for each department, with the column sums equal to the row sums $(X_i)$.

**Table 6 Marx's Reproduction Scheme as an Input-Output Table**

(a) Numerical Representation

| Year 1 | Dept.1 | Dept. 2 | Dept. 3 | $dC$ | $dV$ | $u$ | $X_i$ |
|---|---|---|---|---|---|---|---|
| Dept. 1 | 4000 | 550 | 950 | 500 | | | 6000 |
| Dept. 2 | | | | | | 1100 | 1100 |
| Dept. 3 | 1000 | 275 | 475 | | 150 | | 1900 |
| $S_i$ | 1000 | 275 | 475 | | | | |
| $X_i$ | 6000 | 1100 | 1900 | | | | 9000 |

(b) Algebraic Representation

| Year 1 | Dept.1 | Dept. 2 | Dept. 3 | | |
|---|---|---|---|---|---|
| Dept. 1 | $a_{11}X_1$ | $a_{12}X_2$ | $a_{13}X_3$ | $dC$ | $X_1$ |
| Dept. 2 | | | | $u$ | $X_2$ |
| Dept. 3 | $h_3l_1X_1$ | $h_3l_2X_2$ | $h_3l_3X_3$ | $dV$ | $X_3$ |
| | $S_1$ | $S_2$ | $S_3$ | | |
| | $X_1$ | $X_2$ | $X_3$ | | |

To proceed from an input-output table to a model of input-output relations requires the specification of fixed coefficients. Technical coefficients $a_{ij} = T_{ij} / X_j$ define the ratio between total flows $(T_{ij})$, from department $i$ to department $j$, to gross output $(X_j)$ of department $j$. Similarly, labour coefficients $l_j = L_j / X_j$ define the ratio to gross output of the total number of labour units employed in each sector $(L_j)$. Consumption coefficients $h_i = C_i / L$ can be specified as the ratio to total labour employed $(L)$ of the amount consumed by workers $(C_i)$ of goods produced in department $i$.

Using these coefficients an algebraic representation of the input-output table is displayed in Table 6(b). The relationship between the algebraic and numerical parts of the table can be explained by noting that:

$$a_{11} = \frac{4000}{6000} = \frac{2}{3}, \qquad a_{12} = \frac{550}{1100} = \frac{1}{2}, \qquad a_{13} = \frac{950}{1900} = \frac{1}{2},$$

$$l_1 = \frac{2000}{6000} = \frac{1}{3}, \qquad l_2 = \frac{550}{1100} = \frac{1}{2}, \qquad l_3 = \frac{950}{1900} = \frac{1}{2},$$

$L = 2000 + 550 + 950 = 3500$, and

$$h_3 = \frac{1750}{3500} = \frac{1}{2}.$$

To calculate $a_{12}X_2$ flows of capital goods between departments 1 and 2, for example, we have $½ \times 1100 = 550$. And similarly, the flow of wage goods $h_3l_2X_2$ consumed by workers in department 2 is calculated as $½ \times ½ \times 1100 = 275$.

An input-output model of Table 6, closed with respect to households, takes the form:

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ h_3 \end{bmatrix} \begin{bmatrix} l_1 & l_2 & l_3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} dC \\ u \\ dV \end{bmatrix} \quad (4)$$

or

$$X = AX + hlX + F \quad (5)$$

where $X$ is the column vector of gross outputs for each sector, $A$ is the square matrix of interindustry technical coefficients, h is the column vector of worker consumption coefficients, $l$ is the row vector of labour coefficients, and $F$ is a column vector representing final demand.

To simplify this model, we can now define Y as a column vector of final outputs for each sector, such that $X = AX + Y$, and therefore

$$X = (I - A)^{-1}Y \quad (6)$$

By taking $AX$ to the left-hand side of (5) and substituting (6) it follows that:

$$Y = hvY + F \quad (7)$$

where $v = l(I - A)^{-1}$ is a row vector of vertically integrated labour value coefficients of the type specified by Pasinetti (1981) and Morishima (1973).
Now by pre-multiplying (7) by the vector $v$ such that

$$vY = vhvY + vF \quad (8)$$

and re-arranging, we have:

$$vY = \frac{1}{1 - vh} vF \quad (9)$$

This is an employment multiplier relationship, showing the relationship between final demand $(F)$ and total employment of labour power (vY).

It follows that the value of labour power is represented in the denominator of this employment multiplier by $vh$, the labour required to produce the amount of wage goods consumed by each unit of labour. This term consists of the consumption coefficients $h$ pre-multiplied by

$v = l(I - A)^{-1}$, the vector of labour values. With $vh$ interpreted to be the per capita value of labour power then $1 - vh$ represents the (per capita) share of surplus value e, the amount of surplus value extracted for each unit of labour.6

Since in Marx's reproduction scheme, $v = i'$ 7 it follows that (9) is identical to the income multiplier relationship:

$$y = \frac{1}{e} f \qquad (10)$$

where $y = i'Y$ represents total net income and $f = i'F$ is total final demand. This is a macroeconomic multiplier relationship, which closely resembles the Keynesian multiplier reported in Trigg (2001). Whereas the latter assumed a one-good economy, however, equation (10) is derived from multisectoral foundations.

With total final demand $(f = u + dC + dV)$ made up of investment $(I_v = dC + dV)$ and capitalist consumption $(u)$, equation (10) can be re-expressed as the identity:

$$S = u + I_v \qquad (11)$$

or

SURPLUS VALUE = CAPITALIST CONSUMPTION + INVESTMENT

where $S = ey$ represents the total volume of surplus value produced in the economy.8 Equation (11) provides an alternative way of representing the Kalecki principle in Marx's reproduction scheme. Instead of examining the determinants of gross undistributed profits, as in Kalecki's equation (3), an alternative ex post identity based on the input-output model is derived in which profits (surplus value) are set equal to investment and capitalist consumption. The Kalecki principle, that capitalists earn what they spend, can be applied to equation (11), with capitalist spending on capitalist consumption and investment in constant and variable capital determining the total volume of surplus value. In contrast to the Kalecki formulation there is a clear role for Marx's theory of surplus value. Capitalists cast money into circulation for expenditures in capitalist consumption and investment that are realized as surplus value.

# 4 Conclusions

This paper examines the relationship between Kalecki's macroeconomics and Marx's reproduction schema. With Marx's two-department schema re-cast in a three-department framework, the role of capitalists' personal consumption can be shown explicitly; and by also interpreting Marx's numerical examples as ex post schema, Kalecki's macroeconomic identity can be established between profits and capitalist expenditures on investment and consumption. This result enables the Kalecki principle, that capitalists earn what they spend, to be directly established in the context of Marx's original reproduction tables.

The problem with Kalecki's interpretation, from a Marxian point of view, is that it obscures the role of surplus value in the reproduction schema. An alternative way of identifying the role of the Kalecki principle is provided by Leontief's input-output framework. With Marx's wider definition of investment including increments in constant and variable capital, and by specifying the role of surplus value in the input-output multiplier, an alternative identity between profits and capitalist expenditures is established. This identity enables the Kalecki principle to be represented in the reproduction schema whilst at the same time maintaining the role of surplus value in Marx's system. Capitalists can be argued to cast money into circulation for expenditures in investment and personal consumption, which enable the production and realization of surplus value. Kalecki's macroeconomics can be seen as a way of developing our understanding of Marx's reproduction schema and their relationship to the circulation of money, without necessarily comprising the role of Marx's value categories.

## Appendix -The Structure of the Three-Sector Reproduction Schema

The three-sector reproduction schema in Tables 3 to 5 can be displayed algebraically, showing more precisely the way in which Kalecki's interpretation is derived from Marx's numerical example. Starting with Marx's ex ante scheme, as represented in Table 3, there are three balancing equations:

$$\begin{aligned} C_1 + V_1 + u_1 + dC_1 + dV_1 &= X_1 \\ C_2 + V_2 + u_2 + dC_2 + dV_2 &= X_2 \\ C_3 + V_3 + u_3 + dC_3 + dV_3 &= X_3 \end{aligned} \qquad \text{(A1)}$$

Table 4, the ex post scheme, involves a simple re-arrangement of the elements of each equation such that:

$$\begin{aligned} (C_1 + dC_1) + (V_1 + dV_1) + u_1 &= X_1 \\ (C_2 + dC_2) + (V_2 + dV_2) + u_2 &= X_2 \\ (C_3 + dC_3) + (V_3 + dV_3) + u_3 &= X_3 \end{aligned} \qquad \text{(A2)}$$

In Kalecki's interpretation (Table 5) the equation terms are then grouped according to categories of wages $(W_i^*)$ and profits $(P_i^*)$:

$$\begin{aligned} (V_1 + dV_1) + (C_1 + dC_1 + u_1) &= X_1 \\ (V_2 + dV_2) + (C_2 + dC_2 + u_2) &= X_2 \\ (V_3 + dV_3) + (C_3 + dC_3 + u_3) &= X_3 \end{aligned} \qquad \text{(A3)}$$

where $W_i^* = V_i + dV_i$ and $P_i^* = C_i + dC_i + u_i$.

## References


Brus, W. (1977): "Kalecki's Economics of Socialism", Oxford Bulletin of Economics and Statistics, 39, pp. 57-67.

Cartelier, J. (1996): 'Payment systems and dynamics in a monetary economy' in C. Deleplace and E.J. Nell (eds.), Money in Motion, pp. 200-238, Macmillan, London.

Desai, M. (1979): Maxian Economics, Basil Blackwell, Oxford.

Dixon, R (1988): Production, Distribution and Value: a Marxian approach, Wheatsheaf Books, Brighton.

Freeman, A. and Carchedi, G., (1996): Marx and Non-Equilibrium Economics (eds.), Edward Elgar, Cheltenham.

Kalecki, M. (1936): 'Some Remarks on Keynes's Theory'", in J.Osiatynski (ed.): Collected Works of Michal Kalecki, vol. I, Capitalism: Business Cycles and Full Employment, Clarendon, Oxford, 1990, pp. 223-232.

Kalecki, M. (1968): "The Marxian Equations of Reproduction and Modern Economics", in J.Osiatynski (ed.): Collected Works of Michal Kalecki, vol. II, Capitalist Economic Dynamics, Clarendon, Oxford, 1991, pp. 459-466.

Kerr, P. (1997): "Marx and Kalecki", Contributions to Political Economy, 16, 23-47.

Lee, F.S. (1998): Post Keynesian Price Theory, Cambridge University Press, Cambridge.

Marx, K. (1978): Capital vol. II, 1893, Penguin, London.

Moseley, F. (1998): "Marx's Reproduction Schemes and Smith's Dogma" In Arthur, C.J. and Reuten, G. (eds.), The Circulation of Capital: Essays on Volume Two of Marx's Capital, pp. 159-186, Macmillan, Basingstoke.

Morishima, M. (1973): Marx's Economics, Cambridge University Press, Cambridge.

Olgin, D.S. (1992): "On an Accidental Proof of the 'Fundamental Marxian Theorem'", History of Political Economy, 24.2, pp. 471-475.

Pasinetti, L.L. (1981): Structural Change and Economic Growth: A Theoretical Essay on the Dynamics of the Wealth of Nations, Cambridge University Press, Cambridge.

Reuten, G. (1998): "The Status of Marx's Reproduction Schemes: Conventional or Dialectical Logic?" In Arthur, C.J. and Reuten, G. (eds.), The Circulation of Capital: Essays on Volume Two of Marx's Capital, pp. 187-229, Macmillan, Basingstoke.

Sardoni, C. (1989): “Some Aspects of Kalecki’s Theory of Profits: its Relationship to Marx’s Schemes of Reproduction”, in M. Sebastiani (ed.): Kalecki’s Relevance Today, pp. 206-219, Macmillan, Basingstoke.

Sebastiani, M. (1994): Kalecki and Unemployment Equilibrium, Macmillan, London.

Trigg, A.B. (2001): “Surplus Value and the Keynesian Multiplier”. Review of Radical Political Economics, forthcoming.

## Footnotes

[1] By taking Marx's schema as the starting point, two key assumptions in Kalecki's reproduction schema are not retained in this analysis. First, as Lee (1998) has argued, Kalecki has a Burchardt production model in which each department is vertically integrated, producing its own raw materials. In contrast, Marx assumes that raw materials are a part of constant capital, produced in the first department and circulated to other departments. Second, Sardoni (1989) argues that for Marx capitalists operate at full capacity, in contrast to the Kalecki reproduction schema.

[2] The expression ex ante should not be confused here with Kalecki's (1936) consideration of capitalists' investment decisions. In relation to the reproduction schema, ex ante refers specifically to the imbalance between row and column sums at the start of the production period.

[3] The relationship between Kalecki and the value theory of Marx has been a matter of some dispute. Sebastiani (1994, 108), for example, has argued that Kalecki 'tacitly rejected' the theory of surplus value, whilst Brus (1977, 59) reported that Kalecki felt 'a strong distaste for the Marxian theory of value, which he considered metaphysical…'. In contrast, Kerr (1997, 44) interprets Kalecki's analysis as presuming 'much of Marx's analytical structure and particular theories, … in order to develop the implications of the classical surplus analysis at a more concrete level'. Although Kerr refers to Kalecki's adaptation of Marx's reproduction schema the specific role of surplus value is not explored.

[4] There has been some concern in Marxian circles that the input-output approach imposes physical units of account upon Marx's categories of labour and money (see Moseley 1998). It should be noted, as stated earlier, that the only units of account considered here are money and labour units, and these are assumed to be equivalent. This demonstrates that the input-output approach can provide an improved understanding of Marx's reproduction schema without imposing a physical unit of account.

[5] Each of these terms represents an aggregation of elements across departments, such that $dC = dC_1 + dC_2 + dC_3$, $dV = dV_1 + dV_2 + dV_3$ and $u = u_1 + u_2 + u_3$.

[6] This interpretation of the multiplier, developed here in relation to the three-department model, has been established in relation to the two-department model (Dixon 1988), the one-good Keynesian multiplier (Trigg 2001), and the multisector input-output framework (Olgin 1992).

[7] Since there is an assumed equivalence between money and labour units, the amount of direct labour power employed is equal to the total net income of the economy: $L = vY = i'Y = y$. To prove that $v = i'$ in our numerical example:

$$v = l(I-A)^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 3 & \frac{3}{2} & \frac{3}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

[8] In terms of the numerical example, $e = 1 - vh = 1 - h_3 = 1 - \frac{1}{2} = \frac{1}{2}$ and total net income is equal to $y = L = 3500$. It follows that the total volume of surplus value (see Tables 1 and 6) is calculated by the equation: $S = ey = \frac{1}{2} \times 3500 = 1750$.

***Titles available in the series:***

| | |
|---|---|
| Number 1 | Valuing the environmental impacts of open cast coalmining: the case of the Trent Valley in North Staffordshire<br>*Andrew B Trigg and W Richard Dubourg, June 1993* |
| Number 2 | Scarcity and stability in a very simple general equilibrium model<br>*Vivienne Brown, February 1994* |
| Number 3 | A conflict model, with rational expectations, of the disinflation of the early 1980s<br>*Graham Dawson, February 1994* |
| Number 4 | Foreign Investment, Globalisation and International Economic Governance<br>*Grahame Thompson, May 1994* |
| Number 5 | Testing the Small Country Hypothesis for Developing Countries<br>*Jonathan Perraton, December 1994* |
| Number 6 | The Discovery of 'Unpaid Work': the social consequences of the expansion of 'work'<br>*Susan Himmelweit, June 1995* |
| Number 7 | Exit, Voice and Values in Economic Institutions<br>*Graham Dawson, June 1995* |
| Number 8 | Residential Summer Schools Attendance and Students' Assessed Performances on Open University Foundation Courses<br>*Alan Gillie and Alan Woodley, June 1995* |
| Number 9 | Putting Words into People's Mouths? Economic Culture and its Implications for Local Government<br>*Maureen Mackintosh, December 1995* |
| Number 10 | What is a Fair Wage? A Critique of the Concept of the Value of Labour-Power<br>*Susan Himmelweit, December 1995* |
| Number 11 | The Origin of the Poverty Line<br>*Alan Gillie, December 1995* |
| Number 12 | The Determinants of Product and Process Innovations<br>*Roberto Simonetti, Daniele Archibugi, Rinaldo Evangelista, February 1996* |
| Number 13 | Technical Change and Firm Growth: 'Creative Destruction' in the Fortune List, 1963-1987<br>*Roberto Simonetti, February 1996* |
| Number 14 | Utilities vs. Rights to Publicly Provided Goods: Arguments and Evidence from Health-Care Rationing<br>*Paul Anand and Allan Wailoo, January 2000* |
| Number 15 | Proceeding to the Paddling Pool: The Selection and Shaping of Call Centre Labour<br>*George Callaghan and Paul Thompson, January 2000* |
| Number 16 | Doing 'Qualitative Research' in Economics: Two Examples and Some Reflections<br>*Elizabeth Hill and Gabrielle Meagher, November 1999* |
| Number 17 | Veblen, Bourdieu and Conspicuous Consumption<br>*Andrew B Trigg, January 2000* |

Number 18 The Effect of Idiosyncratic Events on the Feedback between Firm Size and Innovation
*Mariana Mazzucato, January 2000*

Number 19 Non-market relationships in health care
*Maureen Mackintosh and Lucy Gilson, January 2000*

Number 20 Selling pollution and safeguarding lives: international justice, emissions trading and the Kyoto Protocol
*Graham Dawson, October 2000*

Number 21 Entrepreneurship by Alliance
*Judith Mehta and Barbara Krug, September 2000*

Number 22 A disorderly household - voicing the noise
*Judith Mehta, October 2000*

Number 23 Sustainable redistribution with health care markets? Rethinking regulatory intervention in the Tanzanian context
*Maureen Mackintosh and Paula Tibandebage, November 2000*

Number 24 Surplus Value and the Keynesian Multiplier
*Andrew B Trigg, October 2000*

Number 25 Edwards Revised: Technical Control and Call Centres
*George Callaghan and Paul Thompson, November 2000*

Number 26 Social Norms, Occupational Groups and Income Tax Evasion: A Survey In The UK Construction Industry
*Maria Sigala, November 2000*

Number 27 Procedural Fairness in Economic and Social Choice: Evidence from a Survey of Voters
*Paul Anand, December 2000*

Number 28 Alternative rationalities, or why do economists become parents?
*Susan Himmelweit, December 2000*

Number 29 Agglomeration and Growth: A Study of the Cambridge Hi-Tech Cluster
*Suma Athreye, December 2000*

Number 30 Sources of Increasing Returns and Regional Innovation in the UK
*Suma Athreye and David Keeble, January 2001*

Number 31 The Evolution of the UK software market: scale of demand and the role of competencies
*Suma Athreye, September 2000*

Number 32 Evolution of Markets in the Software Industry
*Suma Athreye, January 2001*

Number 33 Specialised Markets and the Behaviour of Firms: Evidence from the UK's Regional Economies
*Suma Athreye and David Keeble, January 2001*

Number 34 Markets and Feminisms
*Graham Dawson, January 2001*

Number 35 Externalities and the UK Regional Divide in Innovative Behaviour
*Suma Athreye and David Keeble, January 2001*

Number 36 Inequality and redistribution: analytical and empirical issues for developmental social policy
*Maureen Mackintosh, March 2001*

Number 37 Modelling the Dynamics of Industry Populations
*Mariana Mazzucato and P A Geroski, January 2001*

Number 38 Advertising and the Evolution of Market Structure in the US Car Industry during the Post-War Period (withdrawn)
*Mariana Mazzucato and P A Geroski, January 2001*

Number 39 The Determinants of Stock Price Volatility: An Industry Study
*Mariana Mazzucato and Willi Semmler, February 2001*