

Copies may be obtained from: Economics Department

Economics Department Faculty of Social Sciences The Open University Walton Hall Milton Keynes MK7 6AA Telephone: 01908 654437 Email: Socsci-economics-support-list@open.ac.uk Fax: 01908 654488

This series is registered under ISSN 1753-2590 (Print) ISSN 1753-2604 (Online)

Economics Research at The Open University

Throughout the 1990s, The Open University has been developing its research capacity in economics. Economists at the OU comprise a lively and expanding group with a wide set of interests ranging from development policy to decision theory, from Marxist theories of profit to libertarian foundations of environmental policy and from econometric analysis of large data sets through institutional economics to the use of case-studies in policy formation. Nearly a 1000 students from around the world register each year to study economics courses and their needs, together with the multi-disciplinary nature of social science at the university, shape out research. Through a variety of personal and group research projects, our work makes a strong contribution to areas like business, public policy and even philosophy where sharply focused analysis can inform decision-making as well as contribute to scientific progress.

In 1999, approximately £250,000 million worth of externally funded grants (3 from the ESRC) were held by discipline members, some of whom also act as consultants to national and international bodies. Approximately half a dozen students are currently reading for doctorates with members of the discipline and we are always interested in proposals from colleagues or potential students who would like to do research with us.

Some of the journals in which discipline members have published include: Annals of Operations Research, Economic Journal, Economica, Economics and Philosophy, Feminist Economics, Feminist Review, International Journal of the Economic of Business, International Journal of Industrial Organisation, Journal of Economic Issues, Journal of Economic Psychology, Journal of the History of Ideas, Journal of Social Policy, Local Government Studies, The Locke Newsletter, Open Learning, Oxford Economic Papers, Public Policy and Administration, Radical Statistics, Revue d' Économie Politique, Risk Decision and Policy, Structural Change and Economic Dynamics, Technovation and Theory and Decision.

The papers contain results of economic research which are the sole responsibility of the authors. Opinions expressed in these papers are hence those of the authors and do not necessarily reflect views of the University.

Pasinetti, Keynes and the Principle of Effective Demand

Andrew B Trigg and Frederic S Lee

June 2003

The Economics Discipline The Faculty of Social Sciences The Open University Walton Hall MK7 6AA A.B.Trigg@open.ac.uk

Department of Economics 211 Haag Hall University of Missouri-Kansas City 5100 Rockhill Road Kansas City, Missouri 64110 USA leefs@umkc.edu

Abstract

This paper explores the relationship between the Keynesian multiplier and Pasinetti's (1981) model of pure production. Key assumptions of Pasinetti's model are its multi-sectoral structure, the definition of all income as a reward to labouring activities, and as a consequence the operation of a pure labour theory of value.

A translation between these models is effected by introducing investment as an exogenous determinant. By drawing from Keynes to apply his concept of the wage unit, it is possible to aggregate from Pasinetti's multi-sectoral model to a genuinely macroeconomic multiplier. For theorists and policy makers, this provides a way of using the Keynesian multiplier without making the restrictive one-commodity assumption. In addition, this formal demonstration adds to the hitherto largely textual literature on the relationship between the wage unit and the labour theory of value.

Finally, this derivation contributes to recent debates between Pasinetti and his critics. Emphasis is placed upon the interpretation of Pasinetti's model as a simplifying device, bringing clarity and precision to economic theory, without making any rash claims about its pre-institutional status. In particular, this approach enables a more precise judgement to be made about which aspects of money are primary or secondary to the principle of demand.

1 Introduction

One of the most ignored and least understood aspects of Keynes's economics is the close attention afforded to problems of aggregation. In writing the *General Theory* (1936) a key difficulty faced by Keynes concerns 'the choice of units of quantity appropriate to the problems of the economic system as a whole'(p.37). He argues that since heterogeneous physical outputs cannot be aggregated, labour should be the unit of account. Dillard (1984) and Wray (1998) interpret Keynes as adopting a labour theory of value that enables a macroeconomic level of analysis in which total employment provides the appropriate index of aggregate output. Instead of using the usual price index in an attempt to measure physical outputs, Keynes divides aggregate money output by the wage unit, the wage rate paid to a unit of unskilled labour.

A possible reason for the suppression of this interpretation of Keynes is the predominance in Keynesian economics of a one-commodity macro economic model. As Kurz (1985) argues, 'The one-commodity model in macroeconomics is generally legitimized in terms of its simplicity' (p. 121). By making this simplifying assumption macro theorists can side-step the problem of aggregation across heterogeneous commodities and ignore the attention paid to this issue by Keynes.

In a bold and incisive attempt to re-introduce multisectoral foundations into Keynesian economics, Pasinetti (1981) proposes a simple alternative to the one-commodity model. Using a pure production model in which there are no social classes and all output is consumed, Pasinetti seeks to provide a methodological starting point for Keynesian economics and other related traditions of economic thought. Instead of assuming a one-commodity world, Pasinetti uses his model to derive a genuinely macroeconomic relationship, which hold regardless of the level of sectoral disaggregation. Core to this relationship is the definition of a macroeconomic condition for full employment under which there must be full expenditure of national income. Pasinetti has generated some controversy (Davidson 2001, Hodgson 1994) by claiming that this condition provides the prebehavioural essence of Keynes's principle of effective demand.

In order to make this approach more understandable to a Keynesian audience, Pasinetti (1997) engages with the Keynesian cross diagram, and its associated multiplier relationship, because of its suitability for illustrating the principle of effective demand. By attempting to strip out of this diagram some of its behavioural properties, Pasinetti argues that the core pre-institutional principle of effective demand can be identified. The problem, however, is that Pasinetti does not make a direct translation between this Keynesian tool of analysis and his own multi-sector model of pure production. If the same core result holds for both models then such a translation should be possible.

The contribution of this article is to establish this translation, thereby providing a more accessible illustration of Pasinetti's (1981) contribution, and showing how the Keynesian model¹ can be derived from multi-sector foundations. Using this translation we demonstrate that the multiplier can embody complex intersectoral relationships which are genuinely macroeconomic but at the same time retain the simplicity of the Keynesian approach. Moreover, based on these analytical foundations a more precise assessment is suggested of the role of money in the principle of effective demand, a contribution that offers some clarity

to recent debates between Pasinetti and his critics. In addition, by making this translation between Pasinetti and Keynes we provide a mathematical formalisation of the hitherto largely textual analysis of the role of wage units and the labour theory of value in Keynes's system. Keynes's method of aggregation, using labour units, is represented in the context of the simple multiplier relationship, re-expressed in terms of its multisectoral foundations.

The paper consists of three parts and a conclusion. In the first part we introduce Pasinetti's multi-sector pure production model, providing a demonstration of the macroeconomic condition for full employment. In the second part, a multi-sector Keynesian multiplier is derived by modifying the pure production model. The role of the wage unit in this multi-sector model is considered in the third part, from which a more accessible derivation of Pasinetti's macroeconomic condition can be established.

2 Pasinetti's Pure Production Economy

Pasinetti (1981) assumes a pure production economy in which labour is the only factor of production used in *m* sectors of production. For each industry *i*, define n_i as the vertically integrated labour coefficient (N_i/Q_i) where N_i is the amount of direct labour required for production of Q_i amount of good *i* together with the indirect labour required to produce intermediate capital goods used in the production of good *i*. In addition, all of the output is consumed by labour, but labour consumes only a part of the good that they directly or indirectly produce. Thus, for each industry *i* we can define c_i as the per capita consumption coefficient (C_i/L) where the total amount of good *i*. The scalar *L* is the quantity of labour employed in the economy as a whole and is equal to $N_1 + ... + N_m$. Now we can set out Pasinetti's (1981) system using the slightly more accessible format reported in a later contribution (Pasinetti 1986). The quantity and price systems take the form:

$$\begin{bmatrix} 1 & 0 & \cdot & \cdot & 0 & -c_1 \\ 0 & 1 & & 0 & -c_2 \\ & & \cdot & \cdot & \cdot \\ & & & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & 1 & -c_m \\ -n_1 & -n_2 & \cdot & \cdot & -n_m & 1 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ \cdot \\ \vdots \\ Q_m \\ L \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(1)

5

It follows from (1) and (2) that

 $Q_i = c_i L, \quad \text{and} \tag{3}$

$$p_i = n_i W \tag{4}$$

with (3) expressing the relationship of physical quantities to labour, and (4) the relationship of prices to the price of labour (w). Pasinetti interprets (3) as representing a demand-determined theory of production, in which the quantity of physical output produced depends upon consumer demand. In addition, (4) demonstrates the working of a pure labour theory of value in which the price of each commodity produced, relative to the price of labour, is proportional to the labour required for its production.

For a non-trivial solution to exist for (1) and (2), the determinant of the coefficient matrix is set equal to zero, such that:

$$\sum_{i=1}^{m} c_i n_i = 1 \tag{5}$$

Equation (5) is interpreted as the condition for full employment if *L* is set equal to L_f , the total amount of labour available for work (see Pasinetti, 1981, p. 32).² This can be explained by examining the binomial term $c_i n_i$. Writing this out explicitly:

$$c_i n_i = \frac{C_i}{L} \frac{N_i}{Q_i} = \frac{N_i}{L} \tag{6}$$

where C_i is the total consumption of good *i*, which is equal to the final quantity of good *i* produced (Q_i) . Since, for a non-trivial solution to be established, these binomial terms must

add up to 1, the proportions of labour employed in each sector must also add up to 1, which means that all labour is fully employed.

Condition (5) also implies that full employment is contingent upon the expenditure of consumers. Using (4), the binomial term can be re-expressed as

$$c_i n_i = \frac{C_i p_i}{wL} \tag{7}$$

Each binomial term represents the proportion of national income wL that is spent as $C_i p_i$ in sector *i*. For full employment to be achieved these proportions must add up to 1, and hence there must be full expenditure of national income. Pasinetti interprets (5) to be the 'Keynesian effective demand condition for full employment' in which there is no savings in the economy as a whole (Pasinetti, 1986, p. 422).

3 Derivation of the Keynesian Multiplier

Pasinetti generated controversy by claiming that his macroeconomic condition for full employment is pre-institutional, a piece of pure economic theory that provides 'sturdy and rocklike theoretical foundations' to the study of more complex economic phenomena (Pasinetti 1994, p. 40). This pre-institutional condition, he argues, provides a fundamental definition of the principle of effective demand, free from behavioural relationships, which Keynes was 'able to perceive, but not explicitly state' (Pasinetti, 2001, p. 384). For Hodgson (1994), however, the economic categories which define the pure production model – prices, wage rates and technical input-output coefficients – are inextricably bound up with a particular type of institutional context. It is difficult to envisage prices and wage rates, for example, without doing so in the context of a market economy, which represents a particular type of institutional arrangement. From a similar perspective, Davidson argued that Keynes defined the principle of effective demand in relation to specific economic behaviour associated with participants in a monetary economy:

The essence of applying the principle of effective demand to a monetary economy lies in the behavioural relationship people display toward money and entrepreneurs display regarding ex ante market profit opportunities'

(Davidson 2001, p. 408).

A particular problem with this debate between Pasinetti and his critics is the inaccessibility of his macroeconomic condition. Compared to the analytical simplicity of Keynesian macroeconomics, the Pasinetti condition, as shown above, is more complex mathematically, requiring the determinant of the coefficient matrix to be set equal to zero. To make his argument more accessible, Pasinetti has in recent years engaged with the Keynesian cross diagram and its associated multiplier relationship because 'there is no need to underline how effective this simple diagrammatic device has been in illustrating one of the basic contributions of *The General Theory*...' (Pasinetti 1997, 97). Despite his stated objective

to provide more clarity in the definition of Keynes's principle of effective demand, the relationship between his interpretation of the Keynesian cross diagram and the pure production model is not clear. Whilst the latter is grounded in multi-sector foundations, the former has the appearance of a sojourn into a qualitatively different modelling framework. In the analysis that follows a more direct translation is suggested between the Pasinetti pure production model and the Keynesian multiplier relationship.

To make this translation a key similarity can be identified between Pasinetti's pure production model and the approach taken by Keynes in parts of the *General Theory*. Like Pasinetti, Keynes assumes that labour is the only factor of production (Reati 2000). Keynes states: I sympathize, therefore, with the pre-classical doctrine that everything is *produced* by *labour*.... It is preferable to regard labour, including, of course, the personal services of the entrepreneur and his assistants, as the sole factor of production...' (1936, pp. 213–14, Keynes's emphasis; see also Naldi, 2000). Using this similarity it is possible to maintain, in a Keynesian system, Pasinetti's association of all income with wages paid to labour.³

In making this translation between the Pasinetti and Keynesian systems two modelling choices can be considered. First, a two-period Austrian model can be assumed. Capital goods are first produced by unassisted labour, and in the second period consumption goods are produced by labour and capital. Conceptually this is the most straightforward way to model a multi-sectoral economy in which labour is the sole factor of production. The derivation of the Keynesian employment multiplier from this model is outlined in the Appendix.

The second alternative, which is developed in the analysis that follows, is to make explicit the circular flow of intermediate commodities that is implicit in Pasinetti's model. In his exposition of the pure production model, Pasinetti (1981, p. 30) is clear that 'it is always possible, when needed, to re-introduce intermediate stages and intermediate commodities by linear algebraic transformations'. This can be achieved by writing equation (1) in block matrix form such that:

$$\begin{bmatrix} I & -c \\ -n & 1 \end{bmatrix} \begin{bmatrix} Q \\ L \end{bmatrix} = \begin{bmatrix} O \\ 0 \end{bmatrix}$$
(8)

where *I* is an $m \times m$ identity matrix, *c* is an $m \times 1$ vector of consumption coefficients, *Q* is an $m \times 1$ vector of final outputs, O is an $m \times 1$ vector of zeros, *n* is a $1 \times m$ vector of vertically integrated labour coefficients, and *L* is the scalar representing the total quantity of labour employed. As shown in Chapter VI of Pasinetti (1981), vertically integrated labour coefficients can be expressed in terms of an explicit input-output structure. In (8) we can reexpress the labour coefficient vector as

$$n = l(I - A)^{-1} \tag{9}$$

where A is an $m \times m$ matrix of interindustry technical coefficients and l is an $1 \times m$ vector of direct labour coefficients, each element l_i representing the direct labour required to produce each unit of gross output in sector i. With X defined as the $m \times 1$ vector of gross outputs, the vector of final outputs can be re-expressed as

$$Q = (I - A)X \tag{10}$$

Substituting (9) and (10) into (8) yields:

$$\begin{bmatrix} (I-A) & -c \\ -l & 1 \end{bmatrix} \begin{bmatrix} X \\ L \end{bmatrix} = \begin{bmatrix} O \\ 0 \end{bmatrix}$$
(11)

In this expression the role of intermediate (circulating) capital inputs is shown explicitly in the (I - A) matrix.⁴

A modifying assumption to the Pasinetti system is required to make the pure production model open with respect to investment. Assume that investment in the current period becomes capital inputs in the next period, where there is 100 per cent depreciation. With M representing an $m \times 1$ vector containing physical quantities of investment goods produced in each sector, Pasinetti's quantity and price systems can be modified to take the form:

$$\begin{bmatrix} (I-A) & -c \\ -l & 1 \end{bmatrix} \begin{bmatrix} X \\ L \end{bmatrix} = \begin{bmatrix} M \\ 0 \end{bmatrix}$$
(12)

$$\begin{bmatrix} p & w \end{bmatrix} \begin{bmatrix} (I-A) & -c \\ -l & 1 \end{bmatrix} = \begin{bmatrix} O & pM/L \end{bmatrix}$$
(13)

where p is an $m \times 1$ row vector of money prices.

In order translate from a pure production model to a Keynesian model, income has to be split up into two categories, the first of which is directed to consumption and the second which is saved and directed to investment by entrepreneurs. Investment activity is financed *ex post* out of savings from wage income. The division of wages between consumption and investment is demonstrated in equation (13) by the pre-multiplication of the row vector of prices by the final column such that:

$$w = pc + pM/L \tag{14}$$

In addition, note that C = cL, where C is the $m \times 1$ column vector containing the total quantities consumed of each sector's output. Multiplying throughout by L:

9

$$Y = Lw = pC + pM \tag{15}$$

Equation (14) shows that the wage rate per unit of labour (w) is made up of total money consumption per unit of labour (pc) plus total money investment per unit of labour (pM/L); equation (15) shows that the entire wage bill (Lw) or national income (Y) equals consumption plus investment. With investment paid for out of wages this maintains the assumption in both Pasinetti and Keynes that all income is paid to labour. It should be emphasised, of course, that the magnitude of investment is determined exogenously by entrepreneurs, but once this magnitude is set, equation (14) determines the wage rate required to finance this expenditure.

It should also be noted that the proportionality between prices and labour values (equation 4) is maintained in this modified Pasinetti framework. Equations representing the first *m* rows and columns of (13) take the form p(I - A) = wl such that:

$$p = l(I - A)^{-1}w$$
 (16)

This derivation explicitly shows the proportionality between prices and the vertically integrated labour coefficients, $n = l(I - A)^{-1}$.

To derive the Keynesian multiplier from this modified Pasinetti framework, the simultaneous equations collected in (12) can be written out such that

 $X = AX + cL + M \tag{17}$

$$L = lX \tag{18}$$

Substituting (18) into (17) yields

 $X = AX + clX + M \tag{19}$

Since from (10), $X = (I - A)^{-1}Q$ it follows that

$$Q = cl(I - A)^{-1}Q + M$$
(20)

or⁵

 $Q = cnQ + M \tag{21}$

10

Multiplying throughout by n:

$$nQ = nc[nQ] + nM \tag{22}$$

and therefore:

$$nQ = \frac{1}{1 - nc} nM \tag{23}$$

This is a multisectoral multiplier relationship between scalars representing the labour required to produce investment goods (nM) and total labour employed (nQ). For the purposes of macro modelling, the multiplier (1/1-nc) is a simple scalar magnitude.⁶ To specify a macro multiplier relationship between total employment and employment in the production of investment goods it is not recessary to specify a one-commodity model. Using Pasinetti's pure production model as a starting point, the macro multiplier relationship can be derived from multisectoral foundations.

4 The Wage Unit and Effective Demand

Subject to certain qualifications Keynes assumes that the employment multiplier also represents an income multiplier (Keynes, 1936, p. 116). By dividing money income by the wage unit, the volume of employment, or labour units, provides an index of real income. This method of aggregation can be formally demonstrated by examining the employment multiplier expression derived in equation (23). Since *L* represents the total units of unskilled undifferentiated labour employed in the economy, then the wage unit represents the money wage paid to each unit of unskilled labour. This wage unit is the same as the wage rate *w* in Pasinetti's model. Furthermore, by assumption, in Keynes and Pasinetti, all money income (Y = pQ) is paid to labour, with profits effectively subsumed as part of wages, the reward to entrepreneurial labour.

The two quantities in the multiplier expression (23) can be written as

$$nQ = \sum_{i=1}^{m} n_i Q_i \tag{24}$$

$$nM = \sum_{i=1}^{m} n_i M_i \tag{25}$$

11

Since, from (4), $n_i = p_i / w$, it follows that

$$nQ = \sum_{i=1}^{m} \frac{p_i Q_i}{w} = Y_w$$
(26)

$$nM = \sum_{i=1}^{m} \frac{p_i M_i}{w} = M_w$$
(27)

By substituting (26) and (27) into (23) the multi-sectoral multiplier relationship can be reexpressed as:

$$Y_w = \frac{1}{1 - nc} M_w \tag{28}$$

This is an income multiplier relationship, with Y_w representing real income (money income deflated by the wage unit), M_w representing real investment (money investment deflated by the wage unit), and *nc* the propensity to consume.⁷ Using some of the shared assumptions of Pasinetti and Keynes the employment multiplier in equation (23) is identical to the income multiplier in (28).⁸

It follows that we have established what Pasinetti refers to as a genuinely macroeconomic relationship, in this case between real income and investment. The structure of this relationship holds regardless of the degree of disaggregation. Although the number of elements of the vectors n and c may vary with the number of sectors, the structure of the relationship remains unchanged. This interpretation of the Keynes multiplier differs markedly from the assumption in Kurz (1985) and much of macroeconomics that the Keynesian multiplier embodies the assumption of a one-commodity world.⁹ Moreover, this multiplier relationship also facilitates a clearer exposition of Pasinetti's principle of effective demand. We have seen that in Pasinetti's pure production model there is no exogenous investment demand, which in a Keynesian macroeconomic system means that $M_w = 0$. Applying this condition to equation (28), and taking the denominator to the left-hand side yields the expression:

$$(1 - nc)Y_w = 0$$
 (29)

This represents a macroeconomic quantity system comparable to Pasinetti's pure production model, which was presented in equation (1). A non-trivial solution requires the identity

$$1 - nc = 0 \tag{30}$$

to be satisfied. This identity is the same as Pasinetti's non-trivial solution in equation 5, that is: $\sum_{i=1}^{m} c_i n_i = 1$. If as before, in our interpretation of equation (5), the total quantity of labour (*L*) is set equal to the total amount of labour available for work (L_f), Pasinetti's effective demand condition for full employment is established using the simple Keynesian multiplier relationship derived from multisectoral foundations

This translation between the Keynesian and Pasinetti systems (equations 28 and 29) provides some clarification of their institutional characteristics. Both systems, as constructed here, can rely on the same multi-sectoral structure and the same configuration of labour inputs and consumption. The *only* mathematical difference is the absence of exogenous investment demand in the Pasinetti system. In terms of institutional structure, of course, this difference is profoundly important. Some of the key characteristics of a monetary production economy, as identified by Paul Davidson (2001), are captured by the existence of a positive volume of exogneous demand, as represented by M_w in equation (28). The main consequence is that there are two types of demand, one representing individual consumption which is dependent upon income and employment, and the other representing investment (M_w) that is not dependent upon income and employment. Once a part of demand is not dependent upon employment '...in the general case, there is no necessity for the determinants of aggregate demand to be identical with other determinants of aggregate supply' (Davidson 2001, p. 393). There is no reason in general why exogenous investment should generate the volume of demand that is required to establish full employment.

If exogenous investment is positive in equation (28) it also follows that *ex post* there must be positive savings out of the national income. In congruence with Keynes's 'fundamental psychological law' (Keynes 1936, p. 96), Pasinetti's condition for full employment (equation 30) must be relaxed, such that the propensity to consume is less than 1. As Keynes states: 'This means that, if employment and hence aggregate income increases, *not all* the additional employment will be required to satisfy the needs of additional consumption' (Keynes, 1936, p. 97, author's emphasis).

On this interpretation there is considerable interface between Pasinetti and Davidson on the key elements of the principle of effective demand. A key area of dispute, however, is the role played by money. Davidson argues that since savings in a money economy are held as non-producible liquid assets this 'can prevent 'saved' ...(i.e., unutilized or involuntarily unemployed) real resources from being employed to expand the economy's productive facilities' (Davidson 2001, p. 400). In Pasinetti's system money also has an important role to play, although Davidson (2001, p.408) inteprets Pasinetti's principle of effective demand as applying to a 'world of barter'. Against this charge, Pasinetti argues that money is in fact required in his multisectoral pure production economy:

An advanced, extremely specialized, production economy, as the one I consider, where each worker's contribution to production can be as specialized as to consist of tiny fractions of a single good or service, could not even subsist as a barter economy. It requires intermediate means to carry out the exchange of services with sums of abstract purchasing power

(Pasinetti 2001, p. 386).

Of possibly more importance is the way in which money is held as savings in the pure production economy. Pasinetti argues (Pasinetti, 1981, p. 35) that single individuals may save while others dissave, but in the aggregate they must cancel each other out. But even though in aggregate there are zero savings, there is a possibility in Pasinetti's system for individuals to save and borrow liquid assets. This has an important consequence for the principle of effective demand. It means that the existence of money that is held as savings by particular individuals in the economy does not in itself provide a defining feature of the principle of effective demand. This behavioural attribute is characteristic of both the pure production and Keynesian systems. The key difference between these systems is whether or not there are positive aggregate savings, as expressed in condition (30).

This interpretation is somewhat consistent with the distinction between a 'neutral' and 'entrepreneur' economy, which Keynes introduces in the 1933 manuscript of the *General Theory* (see Aoki, 2001). A neutral economy is comparable to Pasinetti's pure production model in which money has a facilitative role as a means of payment.¹⁰ Under an entrepreneur economy, where exogenous investment and its impact upon output can be modelled by the Keynesian system, money has complex functions, including its role as a means of debt repayment and speculation. Moreover, the full employment postulates of classical theory, as defined by Keynes, are satisfied in a neutral economy 'provided that the *whole* of current incomes of the factors of production are necessarily spent...' (Keynes, 1979, p. 77, emphasis added). This focus on the importance of zero total savings is captured by Pasinetti's condition for full employment, as defined in equation (30).

In assessing Pasinetti's principle of effective demand, it is advisable not to get distracted by discussing the pre-behavioural status of the pure production model. The task should rather be one of 'minimizing what appears as secondary, and singling out and magnifying what the theorist (in his intuition) identifies as the major or basic element that is capable of providing the explanation that is sought' (Pasinetti 1986, p. 414). For Pasinetti, money can have a facilitative role as a means of payment, and as a store of value in the form of individual savings, with the effective demand condition for full employment still in tact. These aspects of money are secondary to the principle of effective demand. The existence of positive aggregate savings has the primary role for the principle of effective demand, which means 'putting everything else on a secondary, subordinate, position' (Pasinetti 1986, p. 415).

There is no intention, however, to trivialise the role of money in the principle of effective demand. Attendant to there being positive aggregate savings in the Keynesian system, exogenous investment is funded and speculated upon with money playing a key role in a complex network of institutional relationships. The main point to be made is that by isolating the importance of the aggregate savings condition, Pasinetti has developed an analytical foundation for determining which aspects of money have a primary role in the principle of effective demand.

5 Conclusions

Using Pasinetti's pure production model we have derived a simple multiplier relationship that has some resemblance with that used by Keynes in the General Theory. There are three main results from this derivation. First, we show that using Keynes's wage unit a genuinely macroeconomic relationship between real income and investment can be established. At one and the same time this aggregate multiplier relationship embodies inter-sectoral transactions and a pure labour theory of value. As a mathematical derivation, this examination of the multiplier provides a formal representation of recent attempts to interpret Keynes from the perspective of the labour theory of value. Second, this analysis provides a more accessible insight into Pasinetti's principle of effective demand, and this helps to clarify some of the issues raised in recent debates between Pasinetti and his critics. By relating Pasinetti's multisector system directly to the Keynesian multiplier relationship, an aggregate representation is provided of the effective demand condition for full employment. Using this framework it is shown that money has a particular macroeconomic role in Pasinetti's principle of effective demand. Emphasis is placed upon the interpretation of Pasinetti's model as a simplifying device, bringing clarity and precision to economic theory, without making any rash claims about its pre-institutional status.

The final result is that by developing Pasinetti's pure production model it is possible to derive the simple multiplier relationship from multi-sector foundations. Although Pasinetti makes a strong case for using his production model to elucidate the principle of effective demand, macro theorists and policy makers can also make use of the standard Keynesian multiplier without interpreting, for reasons of simplicity, the Keynesian multiplier as a one-commodity model. However, this and the first two results are maintained under Pasinetti's association of all income with wages paid to labour. Whether these results can be maintained in a classbased system in which capitalists receive unearned profits, is an open question and hence the agenda for further work.

Appendix An Austrian Two-Period Production Model

Assume that production in each industry is an Austrian two-period process instead of taking place in a single production period. In the first period a given amount of labour (L_{im}) unassisted produces a given amount of machine or investment good (M_i) : $L_{im} \rightarrow M_i$; then in the second period the machine (M_i) is used (with 100% depreciation) with a given amount of labour (L_i) to produce an amount of consumption good Q_i. Consequently, in the current period of production, there is produced in the economy *m* different machines (or investment goods) and *m* different consumption goods. Thus, the modified Pasinetti quantity and price systems take the following form:

[1	0	0	0	•	•	•	0	0	- <i>c</i>	$a \parallel Q_1$]	Γ	0]				
0	1	0	0				0	0	0			A	I_1				
0	0	1	0				0	0	-c	$\begin{array}{c c} 1 & \mathcal{Q}_1 \\ M_1 \\ 2 & \mathcal{Q}_2 \\ M_2 \end{array}$			0				
0	0	0	1				0	0	0	M_2	2	M	1 ₂				
				•			•	•	•	•			•				
					٠		•	•	•	•			•				
						•	•	•	•	•			•				
0	0	0	0	•	•	•	1	0	$-c_{i}$	$m \parallel Q_m$			0				
0	0	0	0	•	•	•	• 1 0 -7	• 0 1	0	$m \begin{bmatrix} \mathbf{Q}_{m} \\ \mathbf{Q}_{m} \\ \mathbf{M}_{n} \end{bmatrix}$	n	M	1 _m				
$\lfloor -z_1 \rfloor$	$-z_{1m}$	$-z_2$	$-z_{2m}$	•	•	•	$-z_m$	$-z_m$	_n 1	$\int L$			0				
											(A	.1)					
						[1	0 1 0 0	0	0	•	•	•	0	0	$-c_1$	
							0	1	0	0				0	0	0	
							0	0	1	0				0	0	$-c_2$	
							0	0	0	1				0	0	0	
$[p_1]$	$p_{_{1m}}$ •	• •	p_m	P _{mm}		w]					•			•	•	•	
$[P_1]$	P_{1m}		P_m	Pm	п	"1						•		•	•	•	
													•	•	•	•	
							0	0	0	0	٠	٠	•	1	0	$-c_m$	
							0	0 0	0	$0 \\ 0 \\ -z_{2m}$	٠	٠	•	0	1	0	
							$-z_1$	$-z_{1m}$	$-z_{2}$	$-z_{2m}$	•	٠	•	$-z_m$	$-z_{mm}$	1	
r				<i>,</i> ,													

 $= \begin{bmatrix} 0 & 0 & \bullet & \bullet & p_m M/L \end{bmatrix}$

(A2)

where $z_i = L_i/Q_i$;

 $z_{im} = L_{im} / M_i;$

 $p_M = [p_{1m} \quad p_{2m} \quad \bullet \quad \bullet \quad p_{mm}];$ and

 $M = \begin{bmatrix} M_1 \\ M_2 \\ \bullet \\ \bullet \\ M_m \end{bmatrix}$

Wages are allocated between consumption and investment such that:

$$w = pc + p_M M / L \tag{A3}$$

where $p = [p_1 \quad p_2 \quad \bullet \quad \bullet \quad p_m].$

Equation (A3) shows that the wage rate per unit of labour (w) is made up of total money consumption per unit of labour (pc) plus total money investment per unit of labour $(p_M M/L)$.

To derive the Keynesian multiplier from this two-period Austrian model, the expression for total labour in equation (A1),

$$L = z_1 Q_1 + z_{1m} M_1 + \dots + z_m Q_m + z_{mm} M_m$$

can be re-expressed, since $Q_i = c_i L$, such that:

$$L = z_1 c_1 L + z_{1m} M_1 + \dots + z_m c_m L + z_{mm} M_m$$
(A4)

Collecting these terms together in matrix notation:

 $L = zbL + z_M M \tag{A5}$

where $z = \begin{bmatrix} z_1 & z_2 & \bullet & \bullet & z_m \end{bmatrix}$;

$$z_M = \begin{bmatrix} z_{1m} & z_{2m} & \bullet & \bullet & z_{mm} \end{bmatrix}; \text{ and }$$

$$b = \begin{bmatrix} c_1 \\ c_2 \\ \bullet \\ \bullet \\ c_m \end{bmatrix}$$

The employment multiplier can therefore be written as:

$$L = \frac{1}{1 - zb} z_M M \tag{A6}$$

With scalars representing the labour required to produce investment goods $(z_M M)$, total labour employed (L), and the propensity to consume (zb), equation (A6) is a macro multiplier relationship derived from Austrian two-period multi-sectoral foundations. Making use of Keynes's wage unit, equation (A6) can easily be transformed into an income multiplier relationship, the same result established in Section 4.

- Aoki, M. (2001) 'To the Rescue or to the Abyss: Notes on the Marx in Keynes', *Journal* of Economic Issues, Vol. XXXV, No. 4, pp. 931–954.
- Davidson, Paul. (2001) The principle of effective demand: another view, *Journal of Post Keynesian Economics*, Spring, Vol. 23, No. 3, pp. 391–409.
- Dillard, D. (1984) 'Keynes and Marx: a centennial appraisal', *Journal of Post Keynesian Economics*, Vol VI, No.3, pp. 421–431.
- Hodgson, Geoffrey M. (1994) A Comment on Pasinetti. In: R. Delorme and K. Dopfer (Eds.), *Political Economy of Diversity: Evolutionary Perspectives on Economic Order and Disorder*, pp. 46–50. Aldershot, Edward Elgar.
- Keynes, J.M. (1936) *The General Theory of Employment, Interest and Money*. New York, Harcourt Brace.
- Keynes, J.M. (1979) The Collected Writings of John Maynard Keynes. Vol 13, *The General Theory and After: A Supplement*. Edited by D. Moggridge. London, Macmillan.
- Kurz, H.D. (1985) 'Effective Demand in a 'Classical' Model of Value and Distribution: The Multiplier in a Sraffian Framework', *The Manchester School*, pp. 121–137.
- Naldi, N. (2000) 'Keynes on the Nature of Capital: An interpretation of The General Theory's Chapter 16', *Review of Political Economy*, April, Vol. 12, No. 2, pp. 157–169.
- Pasinetti, L.L. (1981) Structural Change and Economic Growth: A Theoretical Essay on the Dynamics of the Wealth of Nations. Cambridge, Cambridge University Press.
- Pasinetti, L.L. (1986) 'Theory of Value A Source of Alternative Paradigms in Economic Analysis', in M. Baranzini and R. Scazzieri (eds.): *Foundations of Economics: Structures of Inquiry and Economic Theory*, Oxford, Basil Blackwell, pp. 409–431.
- Pasinetti, L.L. (1994) Economic theory and institutions. In: R. Delorme and K. Dopfer (Eds.), *Political Economy of Diversity: Evolutionary Perspectives on Economic Order and Disorder*, pp. 34–45. Aldershot, Edward Elgar.
- Pasinetti, L.L. (1997) The principle of effective demand. In: G.C. Harcourt and P.A. Riach (Eds.), *A 'Second Edition' of the General Theory*, vol. 1, pp. 93–104. London and New York: Routledge.
- Pasinetti, L.L. (2001) The principle of effective demand and its relevance in the long run, *Journal of Post Keynesian Economics*, Spring, Vol. 23, No. 3, pp. 383–390.
- Reati, Angelo (2000) The complementarity of the post Keynesian and Marxian paradigms: the case of labour value, *Extraits des Cahiers Economique de Bruxelles*, No. 168.4, pp. 481–510.

- Torr, C.S.W. (1992) 'The dual role of user cost in the derivation of Keynes's aggregate supply function', *Review of Political Economy* 4: 1–17.
- Wray, R. (1998) 'Preliminaries to a Monetary Theory of Production: The Labour Theory of Value, Liquidity Preference and the Two Price Systems', in R. Bellofiore (ed.) *Marxian Economics: A Reappraisal, Volume 1: Method, Value and Money.*

Footnotes

¹ The label 'Keynesian' is used to denote the imperfect overlap between this model and Keynes's system of thought. The objective is to add an additional analytical layer to Pasinetti's system that bears some resemblance with certain aspects of Keynes's system, without making any attempt to essentialise the key aspects of the latter. The principle of effective demand is developed with Pasinetti as the analytical starting point.

² This condition ensures that supply automatically creates its own demand (Say's Law) at all levels of employment. There is no barrier to the achievement of a full employment solution.

³ This does not exclude the possibility that profits could be introduced as a separate category of income, which would possibly push the analysis closer to Keynes (see 1936, pp. 53-4). The objective, however, is to introduce a succinct definition of the principle of effective demand with minimal changes to the Pasinetti pure production model.

⁴ Torr (1992) has examined the relationship between the input-output model and Keynes's definition of user cost.

⁵ In this modified Pasinetti model final output (Q) is made up of consumption (cnQ) and investment (M); in contrast to the original Pasinetti model in which final output and consumption are identical. Equation (10), however, defining the structural relationship between gross and final output, has the same structure in both models.

⁶ This multisectoral structure can be illustrated in a two-sector model in which the key parameters take the form:

$$n = \begin{bmatrix} n_1 & n_2 \end{bmatrix};$$

$$Q = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}; \quad M = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}; \text{ and } c = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}.$$

Hence the elements of the aggregate multiplier relationship are defined as scalars:

$$nQ = n_1Q_1 + n_2Q_2$$
;
 $nM = n_1M_1 + n_2M_2$; and
 $nc = n_1c_1 + n_2c_2$.

⁷ From equation (7),

$$nc = \frac{\sum_{i=1}^{m} C_i p_i}{wL},$$

the ratio of total money consumer expenditures (aggregated across sectors) to total income.

⁸ It has previously been established by Kurz (1985) that the employment and income multipliers are identical when the share of all income is directed to wages, as assumed in the pure production economy.

⁹ In addition to providing insights into the structure of the multiplier, Pasinetti's analysis, which is underpinned by a production model in which the pure labour theory of value is in operation, also shows that the multiplier relationship in equation (28) is perfectly consistent with a labour embodied conception of value. We therefore add to the largely textual analysis of Keynes's writings by Dillard (1984) and Wray (1998) by providing a formal demonstration of the relationship between the wage unit and the labour theory of value.

¹⁰ Keynes also refers to this as a quasi-barter system, since it defines the same full employment outcome as a barter economy. However, pure barter is represented by what Keynes calls a 'cooperative' economy, in which output is distributed without the assistance of money. In contrast to Pasinetti's assertion, considered earlier, that a barter economy could not subsist under a specialised system or relationships, Keynes argues that it could with cooperation between producers.

Titles available in the series:

Number 1	Valuing the environmental impacts of open cast coalmining: the case of the Trent Valley in North Staffordshire Andrew B Trigg and W Richard Dubourg, June 1993
Number 2	Scarcity and stability in a very simple general equilibrium model <i>Vivienne Brown, February 1994</i>
Number 3	A conflict model, with rational expectations, of the disinflation of the early 1980s <i>Graham Dawson, February 1994</i>
Number 4	Foreign Investment, Globalisation and International Economic Governance <i>Grahame Thompson, May 1994</i>
Number 5	Testing the Small Country Hypothesis for Developing Countries Jonathan Perraton, December 1994
Number 6	The Discovery of 'Unpaid Work': the social consequences of the expansion of 'work' <i>Susan Himmelweit, June 1995</i>
Number 7	Exit, Voice and Values in Economic Institutions Graham Dawson, June 1995
Number 8	Residential Summer Schools Attendance and Students' Assessed Performances on Open University Foundation Courses Alan Gillie and Alan Woodley, June 1995
Number 9	Putting Words into People's Mouths? Economic Culture and its Implications for Local Government <i>Maureen Mackintosh, December 1995</i>
Number 10	What is a Fair Wage? A Critique of the Concept of the Value of Labour-Power Susan Himmelweit, December 1995
Number 11	The Origin of the Poverty Line <i>Alan Gillie, December 1995</i>
Number 12	The Determinants of Product and Process Innovations Roberto Simonetti, Daniele Archibugi, Rinaldo Evangelista, February 1996
Number 13	Technical Change and Firm Growth: 'Creative Destruction' in the Fortune List, 1963-1987 Roberto Simonetti, February 1996
Number 14	Utilities vs. Rights to Publicly Provided Goods: Arguments and Evidence from Health-Care Rationing <i>Paul Anand and Allan Wailoo, January 2000</i>
Number 15	Proceeding to the Paddling Pool: The Selection and Shaping of Call Centre Labour <i>George Callaghan and Paul Thompson, January 2000</i>
Number 16	Doing 'Qualitative Research' in Economics: Two Examples and Some Reflections <i>Elizabeth Hill and Gabrielle Meagher, November 1999</i>
Number 17	Veblen, Bourdieu and Conspicuous Consumption Andrew B Trigg, January 2000

Number 18	The Effect of Idiosyncratic Events on the Feedback between Firm Size and Innovation <i>Mariana Mazzucato, January 2000</i>
Number 19	Non-market relationships in health care Maureen Mackintosh and Lucy Gilson, January 2000
Number 20	Selling pollution and safeguarding lives: international justice, emissions trading and the Kyoto Protocol <i>Graham Dawson, October 2000</i>
Number 21	Entrepreneurship by Alliance Judith Mehta and Barbara Krug, September 2000
Number 22	A disorderly household - voicing the noise Judith Mehta, October 2000
Number 23	Sustainable redistribution with health care markets? Rethinking regulatory intervention in the Tanzanian context Maureen Mackintosh and Paula Tibandebage, November 2000
Number 24	Surplus Value and the Keynesian Multiplier Andrew B Trigg, October 2000
Number 25	Edwards Revised: Technical Control and Call Centres George Callaghan and Paul Thompson, November 2000
Number 26	Social Norms, Occupational Groups and Income Tax Evasion: A Survey In The UK Construction Industry <i>Maria Sigala, November 2000</i>
Number 27	Procedural Fairness in Economic and Social Choice: Evidence from a Survey of Voters <i>Paul Anand, December 2000</i>
Number 28	Alternative rationalities, or why do economists become parents? <i>Susan Himmelweit, December 2000</i>
Number 29	Agglomeration and Growth: A Study of the Cambridge Hi-Tech Cluster <i>Suma Athreye, December 2000</i>
Number 30	Sources of Increasing Returns and Regional Innovation in the UK Suma Athreye and David Keeble, January 2001
Number 31	The Evolution of the UK software market: scale of demand and the role of competencies <i>Suma Athreye, September 2000</i>
Number 32	Evolution of Markets in the Software Industry <i>Suma Athreye, January 2001</i>
Number 33	Specialised Markets and the Behaviour of Firms: Evidence from the UK's Regional Economies <i>Suma Athreye and David Keeble, January 2001</i>
Number 34	Markets and Feminisms Graham Dawson, January 2001
Number 35	Externalities and the UK Regional Divide in Innovative Behaviour Suma Athreye and David Keeble, January 2001
Number 36	Inequality and redistribution: analytical and empirical issues for developmental social policy <i>Maureen Mackintosh, March 2001</i>

Number 37	Modelling the Dynamics of Industry Populations Mariana Mazzucato and P A Geroski, January 2001
Number 38	Advertising and the Evolution of Market Structure in the US Car Industry during the Post-War Period (withdrawn) Mariana Mazzucato and P A Geroski, January 2001
Number 39	The Determinants of Stock Price Volatility: An Industry Study Mariana Mazzucato and Willi Semmler, February 2001
Number 40	Surplus Value and the Kalecki Principle in Marx's Reproduction Schema Andrew B Trigg, March 2001
Number 41	Risk, Variety and Volatility in the Early Auto and PC Industry <i>Mariana Mazzucato, March 2003</i>
Number 42	Making visible the hidden economy: the case for gender impact analysis of economic policy Susan Himmelweit, August 2001
Number 43	Learning and the Sources of Corporate Growth Mariana Mazzucato and P A Geroski, June 2001
Number 44	Social Choice, Health and Fairness <i>Paul Anand, September 2002</i>
Number 45	The Integration of Claims to Health-Care: a Programming Approach <i>Paul Anand, November 2002</i>